

## **Hybrid Grey Forecasting Model for Iran's Energy Consumption and Supply**

**Hamidreza Mostafaei**, corresponding author

Department of Economics Energy, Institute for International Energy Studies (Affiliated to Ministry of Petroleum) & Department of Mathematics, Faculty of Science, Shahid Rajaei Teacher Training University, Tehran, Iran. E-mail: hrmostafaei@yahoo.com

**Shaghayegh Kordnoori**

MSc of Statistics & Statistics Expert of Research Institute for ICT, Tehran, Iran.  
Email: sh\_kordnourie@yahoo.com

**ABSTRACT:** Grey theory deals with systems that are characterized by poor information or for which information is lacking. This study presents an improved grey GM (1, 1) model, using a technique that combines residual modification with Markov Chain model. We use energy consumption and supply of Iran to test the accuracy of proposed model. The results show that the Markov Chain residual modification model achieves reliable and precise results.

**Keywords:** Grey Forecasting Model; Markov Chain; Energy System

**JEL Classifications:** C15; C53; C63

### **1. Introduction**

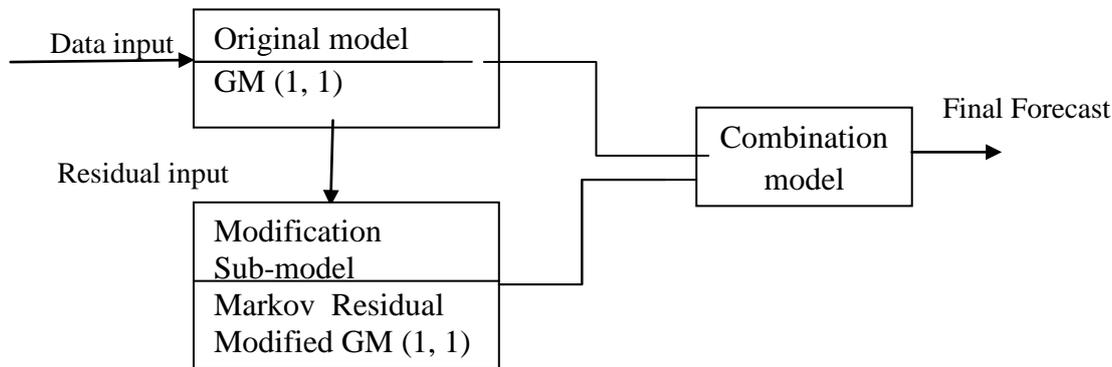
In recent years, grey system theory has become a very useful method of solving uncertainty problems under discrete data and incomplete information. Deng (1982) proposed that the Grey theory concerns with the grey generation, relation analysis, model construction, prediction, decision – making and system control.

Most of the prediction methods require a large number history data and will make use of statistical method to analyze the properties of the system. The statistical predictors may not provide satisfied results owing to the increasing complexity of real-word problems. Furthermore because of additional noise from outside and the complex interrelation among the system or between the system and its environment, it is more difficult to analyze the system. Therefore, methods such as neural network, fuzzy system and grey models are proposed to increase the prediction accuracy. As a prediction model, the grey dynamic model has the advantages of setting up a model with few data.

GM (1, 1) type of grey model is the most widely used in the literature known as Grey Model First Order one Variable. This model is a time series for forecasting model. The differential equations of the GM (1, 1) model have time-varying coefficients. In other words, the model is renewed as the new data become available to the prediction model. This model is easy to understand and simple to calculate with acceptable accuracy, also lack of flexibility to adjust the model to achieve higher forecasting precision. Therefore, researchers begin to shift their attention to find the hybrid Grey model, Grey Markov, Grey Fourier, Grey Fuzzy, etc. Hsu and Chen (2003) proposed an improved grey forecasting model that combines residual modification and residual artificial neural network sign estimation to forecast power demand. Lee and Tong (2011) proposed an approach that combines residual modification and residual genetic programming sign estimation to improve the precision of the residual sign estimator. Hsu et al. (2009) proposed Markov-Fourier grey model prediction approach to predict the turning time of Taiwan weighted stock index for increasing the forecasting accuracy. Kordnoori and Mostafaei (2011) applied the Grey Markov Model for predicting the crude oil production and export of Iran. Hsu (2003) modified the residual of GM (1, 1) model and accepted Markov-Chain sign estimation to forecast the value of the global integrated circuit industry. Yidong

and Hanlin (2010) introduced metabolic grey model together with the residual GM (1, 1) model. Sun and Lin (2009) applied hybrid Grey Forecasting model for Taiwan's Hsinchu science industrial park.

The framework of our research study is as follow:



## 2. Research Methodology

This section introduces how to establish the mathematical model.

### A. To establish GM (1, 1)

Assume an original data series to be  $X^{(0)}(k) = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$ .

The Accumulated Generation Operation (AGO) is expressed as

$$X^{(1)}(k) = \sum_{n=1}^k x^{(0)}(n) \tag{1}$$

The first – order differential equation of GM (1, 1) model is then give as

$$\frac{dX^{(1)}}{dT} + aX^{(1)} = b \tag{2}$$

The solution for (2) is

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a}\right)(1 - e^{-a})e^{-a(k-1)} \quad k = 2, 3, \dots, n \tag{3}$$

Where  $x^{(1)}(1) = x^{(0)}(1)$  and the coefficients  $a$  and  $b$  are called developing and grey input coefficient, respectively. By least-square method, they can be obtained as

$$\begin{pmatrix} a \\ b \end{pmatrix} = (A^T A)^{-1} A^T Y_n \tag{4}$$

Where

$$A = \begin{pmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(n-1) + x^{(1)}(n)) & 1 \end{pmatrix} \tag{5}$$

$$Y_n = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{6}$$

**B. Markov Residual Modified Grey Model (MRMGM)**

Residual errors of Grey Model are obtained using Markov Chain. Markov Chain predicts the future development according to the transition probability among states, which reflects the internal law of all states. Therefore, markov method can be used for predicting of the system with high fluctuation.

Define residual series  $e^{(0)}$  as

$$e^{(0)} = [e^{(0)}(2), e^{(0)}(3), \dots, e^{(0)}(n)] \tag{7}$$

Where  $e^{(0)}(k) = x^{(0)}(k) - \hat{x}^{(0)}(k)$ ,  $k=2, 3, \dots, n$ .

Denote absolute values of residual series as  $\varepsilon^{(0)}$  as

$$\varepsilon^{(0)} = [\varepsilon^{(0)}(2), \varepsilon^{(0)}(3), \dots, \varepsilon^{(0)}(n)] \tag{8}$$

Where  $\varepsilon^{(0)}(k) = |e^{(0)}(k)|$   $k = 2, 3, \dots, n$ .

AGM (1, 1) model of  $\varepsilon^{(0)}$  can be established as

$$\hat{\varepsilon}^{(k)} = \left[ \varepsilon^{(0)}(1) - \frac{b_\varepsilon}{a_\varepsilon} \right] (1 - e^{a_\varepsilon}) e^{-a_\varepsilon(k-1)} \tag{9}$$

Where  $a_\varepsilon, b_\varepsilon$  are estimated using OLS.

Assume that sign of  $k$ th data residual is in state 1 when it is positive and in state 2 when it is negative.

A one step transition probability  $P$  is associated with each possible transition from state  $i$  to state  $j$ , and  $P$  can be estimated using  $P_{ij} = \frac{M_{ij}}{M_i}$ ,  $i, j = 1, 2$ .  $M_i$  means the number of years whose residuals are state  $i$ , and  $M_{ij}$  is number of transitions from state  $i$  to state  $j$  that have occurred.

These  $P_{ij}$  Values can be denoted as a transition matrix R:

$$R = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{10}$$

Denote the initial state distribution by the vector  $\pi^{(0)} = [\pi_1^{(0)}, \pi_2^{(0)}]$ , where  $\pi_1^{(0)}$  and  $\pi_2^{(0)}$  are the transition possibility of state1 and state2. Set the  $n$ th data to be the initial state, and state transition possibility vector to be  $\pi_1^{(0)}$ . Calculation of the state possibility vector of  $(i+1)$ th step transformation after initial state is as follow:

$$\pi^{(i+1)} = \pi^{(0)} R^{(i+1)} \tag{11}$$

Where  $\pi^{(i)}$  are  $k$  th step residual state probabilities.

Let the sign of the  $k$  step residual be represented as follows:

$$\delta(i + 1) = \begin{cases} +1 & \text{if } \pi_1^{(i+1)} > \pi_2^{(i+1)} \\ -1 & \text{if } \pi_1^{(i+1)} < \pi_2^{(i+1)} \end{cases} \tag{12}$$

An improved Grey model with residual modification and Markov Chain sign estimation can be formulated as

$$\hat{x}_r^{(o)}(k) = \hat{x}^{(o)}(k) + \delta(k) \left[ \hat{\varepsilon}^{(o)}(1) - \frac{b_{\varepsilon}}{a_{\varepsilon}} \right] (1 - e^{a_{\varepsilon}}) e^{-a_{\varepsilon}(k-1)} \tag{13}$$

Where  $[\hat{x}_r^{(o)}]^{(1)}(1) = x^{(o)}(1)$  and  $\delta(k) = \pm 1$ .

**C. Error Analysis**

To investigate the accuracy of forecasting models, comparison of forecasting results can be calculated.

Relative Percentage Error (RPE) compares real and forecast values as

$$RPE = \frac{|x^{(o)}(k) - \hat{x}^{(o)}(k)|}{x^{(o)}(k)} \times 100\% \tag{14}$$

**3. Applications**

Energy is an essential source of economic development. Therefore, many countries are concerned with energy –related issues. Energy consumption is an influential economic index, which reflects the industrial development of a country.

In the recent 30 years, the studies on energy system forecasting models have already made great progress, and many forecasting models have been developed. Forecasting energy consumption by common statistical methods usually needs the making of assumptions such as the normal distribution of energy consumption data or a large sample size. However, the data of energy consumption are often very few or non normal. As a grey forecasting model can be formed for at least four date points or ambiguity data, it can be adopted to forecast energy consumption. To minimize the errors of grey forecasting model, we develop an improved grey forecasting model.

Iran ranks among the worlds top tree holders of both proven oil and natural gas reserves. It is one of the leading members of OPEC (Organization of Petroleum Exporting Countries) and the Organization of Gas Exporting Countries (GECF).

To demonstrate the effectiveness of the proposed grey forecasting model, the real case of total energy consumption and supply of Iran is considered as an example. The annual total energy consumption and supply of Iran from 1992 to 2006 are employed as the model – fitting and the data for 2007 and 2008 are utilized as expose testing. The data of this work are listed in table 1, provided by ministry of energy (<http://pep.moe.org.ir>).

**Table 1.** Total energy consumption and supply of Iran from 1992 to 2006.

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
<b>Total energy consumption</b>	496	512	559	561	625	658	680	695	683	682	730	768	831	903	998
<b>Energy supply</b>	630	672	736	771	827	865	897	929	923	933	999	1056	1136	1239	1353

Now we are applying the proposed model to forecast the total energy consumption and supply. From the data in table 1 and the GM (1, 1) model, we obtain:

$$\begin{aligned} \hat{x}^{(o)}(k) &= 497.773e^{0.044378(k-1)} && \text{(Total Energy Consumption)} \\ \hat{x}^{(o)}(k) &= 655.153e^{0.047372(k-1)} && \text{(Energy Supply)} \end{aligned}$$

The absolute values of residual series are:

$$\begin{aligned} \varepsilon^{(o)} &= ( 8.06072, 15.62665, 7.05747, 30.73832, 36.56317, 31.26396, 16.08515, 26.73134, \\ &59.34620, 45.12289, 42.62774, 16.83010, 16.89755, 72.37941) && \text{(Total Energy Consumption)} \\ \varepsilon^{(o)} &= ( 14.53580, 16.23958, 15.79832, 35.26199, 35.24836, 27.07121, 16.84013, 33.93966, \\ &69.76756, 53.14776, 46.88953, 20.70744, 26.27839, 81.54200) && \text{(Energy Supply)} \end{aligned}$$

The Markov Residual Modified Grey Models (MRMGM) is:

$$\hat{x}_r^{(o)}(k) = 497.773e^{0.044378(k-1)} + \delta(k)[29.058 e^{0.008114(k-1)}] \quad k = 2,3, \dots \quad \text{(Total Energy Consumption)}$$

$$\hat{x}_r^{(o)}(k) = 655.153e^{0.047372(k-1)} + \delta(k)[9.80136 e^{0.076182(k-1)}] \quad k = 2,3, \dots \quad \text{(Energy Supply)}$$

As comparing with GM (1, 1) model, the forecast values of 2007 and 2008 by these two methods are listed in table 2 and 3.

**Table 2. The forecast result compared with GM (1, 1) model**

Years	Actual total energy consumption	GM(1,1) model forecast value	precision	MRMGM Forecast precision value
2007	1084.0	968.56	89.35	1001.382 92.38
2008	1115.1	1012.601	90.80	1045.601 93.77

**Table 3. The comparison between GM (1, 1) and MRMGM**

Years	Actual energy supply	GM(1,1) model forecast value	precision	MRMGM Forecast precision value
2007	1453.7	1333.349	91.72	1364.079 93.83
2008	1493.1	1398.033	93.63	1431.196 95.85

The table 2 and 3 show a better precision obtained by the markov residual modified grey model. Finally we predict the total energy consumption and supply from 2009 to 2021 by MRMGM. The forecasted values are listed in table 4.

**Table 4. The predicted values by MRMGM**

Year	Total energy consumption	Energy supply
2009	1091.815	1501.641
2010	1140.117	1575.586
2011	1190.601	1653.204
2012	1243.365	1734.682
2013	1298.513	1820.213
2014	1365.154	1910.003
2015	1416.900	2004.266
2016	1479.368	2103.230
2017	1595.184	2207.131
2018	1613.975	2319.220
2019	1685.877	2430.762
2020	1761.031	2551.033
2021	1839.585	2677.324

#### 4. Conclusion

The original GM (1,1) model is a model with a group of differential equations adapted for variance of parameters. In this article, we have applied an improved grey GM (1,1) model by using a Markov Residual Modified Grey Model. Using the data of total energy consumption and supply of Iran from 1992 to 2006, we concluded that the Markov Chain residual modification model achieves results with higher precisions than the original GM (1,1) model. Finally, we predicted the total energy consumption and energy supply of Iran from 2009 to 2021 by MRMGM.

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